

Letters to the Editor.

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The Loculus of Archimedes.

IN the course of study, of a totally different subject, I recently came across reference to a pastime of the ancient Greeks and Romans, which was of the same nature as the Chinese puzzle, or Tangram, though more elaborate. Curiosity being excited, the scattered references were hunted up, and it was found possible to reconstruct this puzzle with practical certainty, though I have not been able to discover that this has previously been done. From the manner in which this has been received, by those who have seen it, the subject appears to be of sufficient general interest for the result to be placed on record.

To this puzzle the Romans applied the name of *loculus Archimedi*, the first word meaning a small receptacle divided into compartments, the second referring to the name of the reputed inventor. The Greek name is uncertain; the lexicons give it as *ostomachia*, but the only authority quoted is the Latin writer, Ausonius, and this is the form usually adopted by the editors of the printed texts, though the word is sometimes given as *stomachia*. The question of the choice between these two forms was discussed at length by Prof. J. L. Heiberg, in 1907 (*Hermes, Zeitsch. f. class. Philol.*, 12, p. 240), who concluded that the first named, usual, form is a mistaken emendation of the editors, and that the other is the correct one; it is also the form found in a palimpsest, deciphered by him in 1906, and the only known occurrence of the word in a Greek manuscript. If *ostomachia* is correct the interpretation would be a battle or struggle of bones, while *stomachia*, or *stomachion*, would mean the thing that drives one wild.

The descriptions of the Latin writers, being in each case a parenthesis, in illustration of a very different subject, are naturally incomplete, but they show that the puzzle consisted of fourteen pieces of ebony or ivory, mostly in the form of triangles of various kinds and descriptions, with some pieces of a larger number of sides, which were combined to form pictures of fearsome elephants and barking dogs, of ships, castles, and many other objects, but, adds Ausonius, though the compositions of the skilled are wonderful, the efforts of the novice are ridiculous. To this all that can be added is that the whole set was contained in a *forma quadrata*, an expression which, like its English equivalent, square, in geometry implies equality of the four sides, but, in literary or colloquial language, need mean no more than a rectangle.

These descriptions, though they give a good general idea of the puzzle, are no help to a reconstruction of it; of this the first information became generally available in the publication, by H. Suter, of the text and translation of two Arabic manuscripts, preserved in Berlin (*Abh. z. Geschichte d. Mathematik*, 9, 1899, pp. 493-499). The Arabic text is itself a translation from the Greek and is entitled the book of Archimedes on the division of the *Stomachion* into fourteen parts; it contains a description of the construction, followed by a demonstration of the relation of the area of each separate part to the whole. The translation is illustrated by a figure, reproduced in Fig. 1, which appears

to be a reconstruction by the translator, as no mention is made of any figure appearing in the manuscript, and two similar manuscripts, in the India Office and Bodleian libraries, are both wanting in the figure, for which a space is left. The construction may be briefly described: the figure *ABGD*, drawn as a square but described in the Arabic as a parallelogram, is divided into two equal parts by the line *EZ*, parallel to *AB*; the diagonals *AG*, *BZ*, *ZG* are drawn; *AL* is bisected in *M*, and *BM* joined; *BE* is bisected in *H*, from which *HT* is drawn parallel to *AB*, and *HK* as part of the line joining *HA*; *ZG* is bisected in *C*, and *DG* in *N*, and *EC*, *CN* are drawn; finally *CO* is drawn in continuation of *BC*; the whole figure being thus divided into fourteen parts.

This interpretation appears to have been accepted by scholars as a representation of the loculus, but an examination throws doubt on this conclusion. First there is the practical difficulty that it gives a number of very acute angles, which would not only need skilled and careful workmanship to produce, but the razor edges resulting, in one only just over 11°, would be too brittle to stand usage, and would rapidly become damaged. Secondly, a comparison of the translation with the Arabic text shows that while,

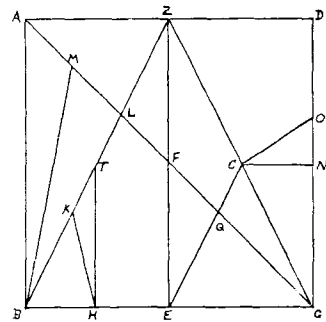


FIG. 1.

in the former, the outer figure *AG* is described as a square, and the two halves *AE*, *ED* as rectangles, the latter consistently uses the same word, which means no more than a parallelogram; and in this the Arabic is logically correct, for it treats the subject as an exercise in pure geometry, and the construction, together with the subsequent discussion, is equally applicable to any parallelogram, irrespective of the magnitude of the sides or angles.

These doubts have been largely cleared up by the discovery, in 1906, by Prof. J. L. Heiberg, of Copenhagen, that a palimpsest, preserved in Constantinople, was overwritten on a copy of the works of Archimedes, which has not only yielded very important additions to the previously known body of his writings, but also contains, at the end, part of a book of the *Stomachion*, the existence of which had been unknown. This fragment was first published in 1913, in the second edition of the works of Archimedes, edited by Prof. Heiberg for the Teubner Classical Library (vol. 2, p. 416 ff.). The opening paragraph of this fragment says that as the thing called the *Stomachion* presents examples of the method of transposition of figures, he thinks it well to treat of these and to show how it is divided and how the parts resemble each other, so that, in forming them into pictures, one may see how the angles may be combined to form two right angles, and whether two sides, which appear to lie in a straight line, do so, or depart slightly from it; though a picture is not necessarily to be rejected on account of a small gap resulting from this. It is evident from this

summary that the complete book would have made a very interesting treatise on the game, but only two short passages remain. The first, following immediately on the introduction, demonstrates that the angle AMB (Fig. 2) must be an obtuse one, and the second, following on a gap, is part of the description of a construction, similar, so far as it goes, to that

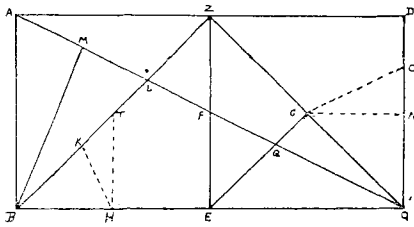


FIG. 2.

described in the Arabic manuscript, with the instructive addition that, after the first division of the whole figure into two equal halves, each half is a square.

From these passages two conclusions may be drawn: that the pastime was really an invention of Archimedes, and not merely fathered on him, and that the initial figure of the loculus was not a square, but a rectangle composed of two squares, set side by side. This makes it possible to reconstruct the pattern of the divisions, as shown in Fig. 2, where the firm lines show the construction, so far as the text of the

triangles, based on this line, and the angle between them can readily be matched, as can the combined triangle ABL. The other exception is the triangle OCN, and this is a very different case; if drawn according to the Arabic construction, with CO as a continuation of BC, it would have the angles of the diagonal of a rectangle of sides three by one, and the common side CO, the two sides NO, OD, and the four angles at O and C would none of them be matched with any other sides or angles. If, however, DN is bisected in O, we have a construction more in tenor with the rest, and get a triangle with angles of the diagonal of a two by one rectangle, though the length of the side CO, being one quarter of AG, would still be impossible to match with any other side. For these reasons it seems probable that, whatever may have been the original construction, designed as an exercise in geometry, the modification would soon have been introduced when it was adopted as a pastime, for, so far as may be judged from a slight experience, it is a marked improvement.

However this may be, it seems clear that, with this possible, though not probable, exception, we have, in Fig. 2, the actual pattern of the pieces forming the loculus mentioned in Latin literature. As examples of what can be done with it I may give a few ridiculous efforts of a novice (Fig. 3); the elephant (i), the ship (ii) and the two figure studies, which I owe to Sir Richard Paget, of the Pied Piper (iii) and the Pundit (iv) will give some indication of the wide range of subjects which can be depicted, in a somewhat cubist

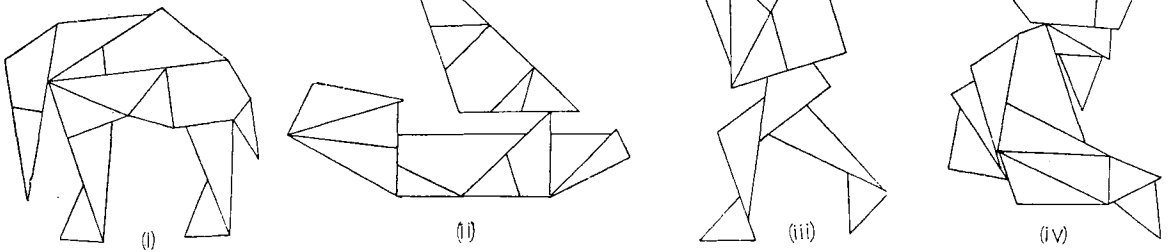


FIG. 3.

palimpsest goes, and the dotted lines the completion, taken from the Arabic text. I have, however, made one departure from the Arabic, in drawing the triangle OCN, which appears necessary if the thing is to be used in the manner described, and not as a mere exercise in geometry.

If Fig. 2 is examined it will be seen that the angles are either right angles, or the angles made by the diagonal of a square, or those made by the diagonal of a rectangle twice as long as it is broad, or the sum or difference of some two of these, so that there are many possible combinations by which either two or four right angles can be built up. Also, for length of side, we have the whole, half or quarter of AB, the half, third or sixth of BZ, and the third or sixth of AG, so that there are a number of sides which can be matched with each other, either singly or in combination. To this there are two exceptions: the line BM cannot be matched with any other side or combination of sides, nor can the angles at B and M be matched with any others to complete either two or four right angles, but the other sides of the two

style it is true. The last-named figure, it may be noticed, contains one of those small gaps, which, the palimpsest expressly declares, need not involve the rejection of the figure. In each case the whole fourteen pieces are employed, this being, presumably, of the essence of the game; it happens at times that a satisfactory figure may be built up, with one or two pieces to spare, and then the fitting in of these superfluous pieces becomes the thing that drives one wild.  
R. D. OLDHAM.

**Use of an Artificial Horizon in Photographic Measurements of Buildings or other Structures.**

It is sometimes required to know whether certain lines intended to be vertical or horizontal in buildings or other structures are really so, and if not, by how much they deviate from these directions. All such measures can, of course, be made by a theodolite or level, but when many details have to be examined this is a long process.